Investigating 3D features of an intermittent cavity flow
experimental and numerical analysis

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Coherent structures

Even if both open and confined flows are complex, and has potentially an infinite number of DoF, coherent structures (vortex, soliton, saturated instabilities) are present and it looks like it drives the dynamics.

It seems legit to look after a 'low' number of modes.

Modal decomposition(s)

Suppose space and time separation and assume the existence of a basis, \( \{ \psi_k(r) \} \) or \( \{ \alpha_k(t) \} \), to describe any flow realization

\[
\mathbf{v}(r,t) = \sum_{k \geq 1} \alpha_k(t) \psi_k(r) \approx \sum_{k=1}^{N_m} \alpha_k(t) \psi_k(r)
\]

→ Proper Orthogonal Decomposition \( (\alpha_k(t), \psi_k(r)) \)
→ Global modes \( (\omega, \psi_\omega(r)) \)
→ Dynamical modes \( (\omega, \psi_\omega(r)) \)
Outline

1 Modal decomposition
   - Dynamic Mode Decomposition (DMD)
   - Proper Orthogonal Decomposition (POD)
   - Spatial properties inheritance

2 Investigation of flow features
   - Comparison 2D POD/2D DMD
   - Divergence analysis
   - Comparison 2D DMD modes vs 3D DMD modes

3 Conclusion
Dynamic Mode Decomposition
What are dynamic modes?


→ Assume there exists an operator of evolution, $A$, such as the $v_k$ are realisations of a nonlinear process.

→ Find a similar matrix to $A$. **Dynamic modes are defined as eigenvectors of** $A$, computed thanks to the similar matrix.
How to compute DMD modes?

With $\mathbf{v}_{N+1} = \mathbf{v}_1 \mathbf{c}_1 + \ldots + \mathbf{v}_N \mathbf{c}_N$, then:

$$AK_1^N = K_1^N \mathbf{C} = K_2^{N+1} \Rightarrow \mathbf{C} = \begin{pmatrix}
0 & 0 & \ldots & 0 & c_1 \\
1 & 0 & \ldots & 0 & c_2 \\
0 & 1 & \ldots & 0 & c_3 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1 & c_N
\end{pmatrix}$$

Eigenvalues of $\mathbf{C}$ are eigenvalues of $\mathbf{U}$. Let $\nu$ be an eigenvector associated with the eigenvalue $\lambda$:

$$\mathbf{U}K_1^N \nu = K_1^N \mathbf{C} \nu = K_1^N \lambda \nu$$

$$\mathbf{U} \left( K_1^N \nu \right) = \lambda \left( K_1^N \nu \right)$$

Eigenvectors $\psi$ of $\mathbf{U}$ are derived from eigenvectors $\nu$ of $\mathbf{U}$: $\psi \equiv K_1^N \nu$
A power spectrum can be constructed on $\lambda = \| \lambda \| e^{-\sqrt{-1}2\pi dt}$ and $\| \psi \|$.
Proper Orthogonal Decomposition
A well-known method

We look for an orthonormal basis of spatial modes \( \{ \psi_i \} \), called **topos**, and temporal modes \( \{ \alpha_i \} \), called **chronos**, such as the average least-squares truncation error,

\[
    r_m = \sum_{k=0}^{t_N} \left\| \mathbf{u}(r, t_k) - \sum_{i=0}^{m} \alpha_i(t_k) \psi_i(r) \right\|,
\]

Chronos and topos are obtained through an Singular Values Decomposition of the dataset.

M. Bergmann, L. Cordier, JP. Brancher, (2007), *NNFM*
Spatial properties inheritance
Example on the divergence criterion

Spatial properties of the flow

$\nabla \cdot \mathbf{v}(r,t) = 0$

By injecting the modal decomposition:

\[
\nabla \cdot \mathbf{v}(r,t) = \nabla \cdot \left( \sum_i \alpha_i(t) \Phi_i(r) \right) \\
= \sum_i \alpha_i(t) \nabla \cdot \Phi_i(r) \\
= 0.
\]

Then, remembering $\alpha_j(t)$ form an orthonormal basis, we have:

\[
\int_{-\infty}^{\infty} \alpha_j(t) \sum_i \alpha_i(t) \nabla \cdot \Phi_i(r) \, dt = \int_{-\infty}^{\infty} \sum_i \alpha_j(t) \times \alpha_i(t) \nabla \cdot \Phi_i(r) \, dt \\
= \sum_i \nabla \cdot \Phi_i(r) \int_{-\infty}^{\infty} \alpha_j(t) \times \alpha_i(t) \, dt \\
= \nabla \cdot \Phi_j(r) \\
= 0.
\]

Henceforward, as for the observable $\mathbf{v}$, the divergence of each mode is zero.
Cavity flow
Cavity flow

Cavity flow: Experimental setup

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Cavity flow

Cavity flow : Experimental setup

- Cavity length : $L = 100$ mm
- Geometric ratio : $L/H = 2$
- Incoming velocity : $U = 1.90$ m/s
- Dataset : $N = 4096$ velocity fields.
- $Re_L = UL/\nu_{\text{air}} = 12700$
- Dominant frequencies in the flow $St_L = fL/U \propto 1$ ($\approx 20$Hz)
- Sampling frequency : 250Hz
Proper Orthogonal Decomposition analysis
POD analysis

POD on the flow dataset

Topos n° 4

Topos n° 2

Topos n° 3

Topos n° 5

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Dynamic Modes Decomposition analysis
Cavity DMD

DMD on the flow dataset

St = 0.08

Spectra

St = 0.13

St = 1.01

St = 1.38

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POD-DMD comparison
POD vs DMD

Detection of intermittency

sliding DMD

sliding Fourier analysis on a chronos

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POD vs DMD

POD mixes lengths

\[
\lambda_{POD}/L = \frac{\gamma_1 \lambda_1/L + \gamma_2 \lambda_2/L}{\gamma_1 + \gamma_2},
\]

where \(\gamma_1\) and \(\gamma_2\) are the respective amplitudes of DMD modes in the spectrum.
POD vs DMD

POD mixes lengths

**Table:** Frequency and wavelength comparison between POD analysis and DMD analysis

<table>
<thead>
<tr>
<th>POD dominant mode</th>
<th>St</th>
<th>λ/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMD mode Φ₁</td>
<td>1.02</td>
<td>0.49</td>
</tr>
<tr>
<td>DMD mode Φ₂</td>
<td>1.38</td>
<td>0.38</td>
</tr>
</tbody>
</table>

POD is not fitted for analysis of intermittency!

\[ \lambda_{POD}/L = \frac{\gamma_1 \lambda_1/L + \gamma_2 \lambda_2/L}{\gamma_1 + \gamma_2}, \]

where \( \gamma_1 \) and \( \gamma_2 \) are the respective amplitudes of DMD modes in the spectrum.
Divergence analysis
Divergence

Divergence of main DMD modes

$St = 0$

$St = 0.1$

$St = 0.3$

$St = 1.0$

$St = 1.4$

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3D DMD modes
3D DMD

Comparison with shear layer DNS DMD mode
3D DMD

Comparison with inner flow DNS DMD mode
Conclusions

- Modes inherit spatial properties of the flow
- Application of DMD to an intermittent flow
- 3D features inferred from 2D experimental dataset
- Confirmation with a 3D DMD analysis on a numerical dataset
This is the end

Thank you for your patience and attentiveness!