Mathematics for Engineers I
Introduction to linear algebra : Matrices

F. Guéniat, V. Venkatesh

florimond.gueniat@bcu.ac.uk

Week Eight

Birmingham City University, Engineering and Built Environment

BIRMINGHAM CITY University

What will you learn

These lessons will be mostly, obviously, maths :

- what is *matrices*
- how it works - practice !
- why it is useful - physics and engineering

I will try to explain in detail (maybe too much). *Please* let me know if you do not understand something.

What you will have to remember

- what is a matrix
- matrix algebra
  - addition, substraction
  - multiplication
- determinent and minors for
  - 2×2 matrices
  - 3×3 matrices
- solving linear systems with matrices

Why bothering ?

Matrices are virtually everywhere :
Electrical engineering :
matrices are used for studying :

- Kirchoff's laws - both voltage and current
- resistor conversion

Why bothering ?

Matrices are virtually everywhere :
Mechanical engineering :
matrices are used for studying :

- stress in materials
- mesh of beams
- mass matrix

Why bothering ?

Matrices are virtually everywhere :
Engineering in general :
matrices are used for :

- modeling (from data or linear models)
- solving the models (eigenproblems and Krylov algorithms)
- calculations

Why bothering ?

Matrices are virtually everywhere :
Computer science :
matrices are used for :

- transformations (VG, RV etc.)
- visualization (medical applications)
- calculations in general

Definition of a matrix

A *matrix* is an array of numbers (or symbols) arranged in *rows and columns*, similarly to a table or a spreadsheet, for instance :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix}$$

Matrices are usually noted with a capital letter.

Definition of a matrix

A *matrix* is an array of numbers (or symbols) arranged in *rows and columns*, similarly to a table or a spreadsheet, for instance :

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 7 & 8 & 9 \\ 1 & 2 & 5 \end{pmatrix}$$

Matrices are usually noted with a capital letter.

Definition of a matrix

A *matrix* is an array of numbers (or symbols) arranged in *rows and columns*, similarly to a table or a spreadsheet, for instance :

$$C = \begin{pmatrix} 1.1 & -5.9 & 3.0 & -1.8 \\ -1 & 7.6 & 3.2 & -1.7 \\ 1.8 & -4.1 & 5.2 & 0 \end{pmatrix}$$

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Definition of a matrix

A *matrix* is an array of numbers (or symbols) arranged in *rows and columns*, similarly to a table or a spreadsheet, for instance :

$$D = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

Matrices are usually noted with a capital letter.

Elements of a matrix

The numbers or symbols are called elements.
For instance, the matrix A :

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

has four elements : $1, 2, 3$ and 4.

Elements of a matrix

If we have A

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

The *elements* of a matrix A can be noted :

- $A(1,2) = 2$ *used in computer science*
- $a_{12} = 2$ *used in maths*
- $A_{12} = 2$ *used in engineering*

Here, in a_{12}, the 1 notes the first line, and 2 corresponds to the second column.
It is always *row first then column*.

Example

If we have B

$$B = \begin{pmatrix} -3.3 & 3.1 & 9.4 \\ 1.4 & 0.9 & -2.6 \end{pmatrix}$$

Then, we have :

	first column	second column	third column
first row	$b_{11} = -3.3$	$b_{12} = 3.1$	$b_{13} = 9.4$
second row	$b_{21} = 1.4$	$b_{22} = 0.9$	$b_{23} = -2.6$

Dimensions of a matrix

The dimensions p, q of a matrix are the numbers of lines p and the number of columns q :

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

The dimensions of A are $2, 2$, or A is a 2×2 matrix.

Always start with the number or rows.

Dimensions of a matrix

The dimensions p, q of a matrix are the numbers of lines p and the number of columns q :

$$B = \begin{pmatrix} -3.3 & 3.1 & 9.4 \\ 1.4 & 0.9 & -2.6 \end{pmatrix}$$

The dimensions of B are $2, 3$, or B is a 2×3 matrix.

Always start with the number or rows.

Dimensions of a matrix

The dimensions p, q of a matrix are the numbers of lines p and the number of columns q :

$$C = \begin{pmatrix} 1 & 9 \\ -3 & 1 \\ 4 & -2 \\ 1 & -6 \end{pmatrix}$$

The dimensions of C are $4, 2$, or C is a 4×2 matrix.

Always start with the number or rows.

Square or rectangle ?

A *square* matrix has the *same number of rows and columns*.
If a matrix is not square, it is rectangular.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

A is square

Square or rectangle ?

A *square* matrix has the *same number of rows and columns*.
If a matrix is not square, it is rectangular.

$$B = \begin{pmatrix} -3.3 & 3.1 & 9.4 \\ 1.4 & 0.9 & -2.6 \end{pmatrix}$$

B is rectangular

Square or rectangle ?

A *square* matrix has the *same number of rows and columns*.
If a matrix is not square, it is rectangular.

$$C = \begin{pmatrix} 1 & 9 \\ -3 & 1 \\ 4 & -2 \\ 1 & -6 \end{pmatrix}$$

C is rectangular

Square or rectangle ?

A *square* matrix has the *same number of rows and columns*.
If a matrix is not square, it is rectangular.

$$D = \begin{pmatrix} 2 & 0 & -1 \\ -3 & 4 & -2 \\ -1 & 2 & -4 \end{pmatrix}$$

D is square

Transpose of a matrix

The transpose N of a matrix M, also noted M^T, is a matrix where the rows of N correspond to the columns of A.

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Mathematically, we have : Let $M = (m_{ij})$ an $n \times m$ matrix.
$N = (n_{ij})$ is the transpose of M if :

- N is an $m \times n$ matrix
  dimensions are switched !
- for any $1 \leq i \leq m$ and for any $1 \leq j \leq n$, $n_{ij} = m_{ji}$
  rows of M $\rightsquigarrow$ columns of N !

Transpose of a matrix

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  rows of $M \rightsquigarrow$ columns of N !

For instance,

$$\text{if } B = \begin{pmatrix} -3.3 & 3.1 & 9.4 \\ 1.4 & 0.9 & -2.6 \end{pmatrix} \text{ then } B^T = \begin{pmatrix} -3.3 & 1.4 \\ 3.1 & 0.9 \\ 9.4 & -2.6 \end{pmatrix}$$

Multiplication by a scalar

Multiplying by a scalar a matrix is simply multiplying every elements of this matrix by the scalar. If

$$A = [a_{ij}]$$

then

$$c \times A = [c \times a_{ij}]$$

.

Multiplication by a scalar

For instance,
if $c = 2.5$ and

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}$$

then :

$$c \times A = \begin{pmatrix} 2.5 \times 1 & 2.5 \times (-3) \\ 2.5 \times 4 & 2.5 \times 2 \end{pmatrix} = \begin{pmatrix} 2.5 & -7.5 \\ 10 & 5 \end{pmatrix}$$

Multiplication by a scalar

For instance,
if $c = -2$ and

$$B = \begin{pmatrix} -3.3 & 3.1 & 9.4 \\ 1.4 & 0.9 & -2.6 \end{pmatrix}$$

then :

$$c \times B = \begin{pmatrix} -2 \times (-3.3) & -2 \times 3.1 & -2 \times 9.4 \\ -2 \times 1.4 & -2 \times 0.9 & -2 \times (-2.6) \end{pmatrix}$$

and then

$$c \times B = \begin{pmatrix} 6.6 & -6.2 & -18.8 \\ -2.8 & -1.8 & 5.2 \end{pmatrix}$$

Considering Addition, Subtraction, Equality

Let's consider two matrices A and B.

For

- addition
- subtraction

the dimensions of A have to be equal to the dimensions of B !

Considering Addition, Subtraction, Equality

Let's consider two matrices A and B.

For

- addition
- subtraction

the dimensions of A have to be equal to the dimensions of B !

Before anything else, you have to check the size of the two matrices !

Addition and subtraction

If A and B has the same dimensions, then they can be added, and the resulting matrix C is :

$$C = A + B$$

any element c_{ij} of C is the sum of the corresponding elements a_{ij} of A and b_{ij} of B :

$$c_{ij} = a_{ij} + b_{ij}$$

Addition and subtraction

If A and B has the same dimensions, then they can be added, and the resulting matrix C is :

$$C = A + B$$

any element c_{ij} of C is the sum of the corresponding elements a_{ij} of A and b_{ij} of B :

$$c_{ij} = a_{ij} + b_{ij}$$

For subtraction, we first multiply by -1 the matrix that we want to subtract.

$$C = A + (-B)$$

any element c_{ij} of C is the sum of the corresponding elements a_{ij} of A and $-b_{ij}$ of $-B$:

$$c_{ij} = a_{ij} - b_{ij}$$

Addition and subtraction

If

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}, B = \begin{pmatrix} 4 & -2 \\ 5 & -1 \end{pmatrix}$$

and $C = A + B$, then :
$c_{11} = a_{11} + b_{11}$, $c_{12} = a_{12} + b_{12}$, $c_{21} = a_{21} + b_{21}$ and $c_{22} = a_{22} + b_{22}$:

$$C = A + B = \begin{pmatrix} 1+4 & -3+(-2) \\ 4+5 & 2+(-1) \end{pmatrix} = \begin{pmatrix} 5 & -5 \\ 9 & 1 \end{pmatrix}$$

Addition and subtraction

If

$$C = \begin{pmatrix} 1 & 5 & 8 \end{pmatrix}, D = \begin{pmatrix} 0 & -3 & 1 \end{pmatrix}$$

then

$$C - D = \begin{pmatrix} 1-0 & 5-(-3) & 8-1 \end{pmatrix} = \begin{pmatrix} 1 & 8 & 7 \end{pmatrix}$$

Addition and subtraction

If

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}, E = \begin{pmatrix} 1 & 1.5 \\ -1 & -3 \\ 1.5 & 2 \end{pmatrix}$$

then we can not add nor subtract A and E, as their size are incompatible !

Equality

Two matrices A and B are equal

- if all their elements are identical
- or if $A - B$ is a matrix with only 0 as elements.

It means that A and B have to have the same dimensions.

Equality

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}$$

are equal.

Equality

$$C = \begin{pmatrix} 1 & 5 & 8 \end{pmatrix}, D = \begin{pmatrix} 1 & 5 & 8 & 0 \end{pmatrix}$$

C and D are *not* equal :

- the dimensions of C are 1×3
- the dimensions of D are 1×4

Equality

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}, E = \begin{pmatrix} 1 & -3 \\ 4 & 2 \\ 1 & 1 \end{pmatrix}$$

A and E are *not* equal :

- the dimensions of A are 2×2
- the dimensions of D are 3×2

Equality

$$F = \begin{pmatrix} -9 & 7 & 0 \\ 3 & 1 & 4 \\ -3 & 2 & 1 \end{pmatrix}, G = \begin{pmatrix} -9 & 7 & 0 \\ 3 & 1 & 4 \\ -3 & 2 & 1 \end{pmatrix}$$

are equal.

Equality

$$F = \begin{pmatrix} -9 & 7 & 0 \\ 3 & 1 & 4 \\ -3 & 2 & 1 \end{pmatrix}, H = \begin{pmatrix} -9 & 7 & 1 \\ 3 & 1 & 4 \\ -3 & 2 & 1 \end{pmatrix}$$

are *not* equal, as $f_{13} = 0$ and $g_{13} = 1$, and hence

$$f_{13} \neq g_{13}$$

What we have seen so far

So far, manipulation matrices is the same as manipulating numbers. In that regard, matrices are just "*bigger*" numbers.
In particular, regular algebra rules hold :

- $A + (B + C) = (A + B) + C$
- $k \times (A + B) = k \times A + k \times B$

However, we will see that multiplication does not work that nicely.
=(

Multiplication : check the size

For multiplying matrices, once again, we have to check dimensions.
But the dimensions do *not* have to be equal, just *compatible*.

> The *second dimension* of the *first matrix*
> *the columns*
> is equal to
> The *first dimension* of the *second matrix*
> *the rows*

If we want to calculate $A \times B$, then the number of columns of A has to be equal to the number of rows of B.

Multiplication : check the size

If

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}$$

we can calculate $A \times B$ and $B \times A$.

Multiplication : check the size

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & -3 \\ 4 & 2 \\ 1 & 1 \end{pmatrix}$$

- the dimensions of A are 2×2
- the dimensions of C are 3×2

The dimensions of C and A are compatible : we can calculate $C \times A$.
The dimensions of A and C are *not* compatible : we can *not* calculate $A \times C$.

It means that $A \times B$ and $B \times A$ are most of the time different !

$$A \times B \neq B \times A$$

Multiplication : check the size

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}, D = \begin{pmatrix} -9 & 7 & 0 \\ 3 & 1 & 4 \\ -3 & 2 & 1 \end{pmatrix}$$

We can neither calculate $A \times D$ nor $D \times A$, their dimensions are incompatible.

If we have $A = (a_{ij}$ is an $n \times p$ matrix and if $B = (b_{ij})$ is an $p \times m$ matrix, then we can multiply them as $A \times B$.
The resulting matrix $C = A \times B$ is :

- an $n \times m$ matrix
- $C = (c_{ij})$, with the formula to calculate $c_{ij} = \sum_{k=1}^{p} a_{ik} \times b_{kj}$

It is a bit strange, right ?

multiplication : how-to

If $C = A \times B$:

- You chose a row i and a column j
  1. You pair the first element of the row i of A (it is a_{i1}) with the first element of the column j of B (it is b_{1j}).
  2. You multiplying them : $a_{i1} \times b_{1j}$.
  3. You pair the second element of the row i of A with the second element of the column j of B.
  4. You multiplying them and add them to the result of 2 : $a_{i1} \times b_{1j} + a_{i2} \times b_{2j}$
  5. You continue until the end of the row.
     this is why the second dimension of A has to be equal to the first dimension of B
- c_{ij} is the result
- you chose another row and column.

Visual example

If $A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$, then

$C = A \times B = \begin{pmatrix} 1 & -8 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ which can be found as :

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -8 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

$$-1 \times 3 + 2 \times 2 = 1$$

Visual example

If $A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$, then

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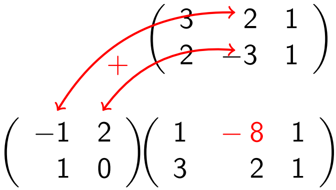

$$-1 \times 2 + 2 \times -3 = -8$$

Visual example

If $A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$, then

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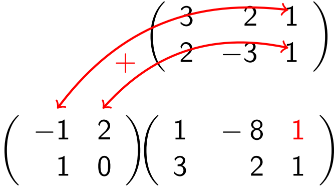


$$-1 \times 1 + 2 \times 1 = 1$$

Visual example

If $A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$, then

$C = A \times B = \begin{pmatrix} 1 & -8 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ which can be found as :

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -8 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

$$1 \times 3 + 0 \times 2 = 3$$

Visual example

If $A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$, then

$C = A \times B = \begin{pmatrix} 1 & -8 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ which can be found as :

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$$

$+$

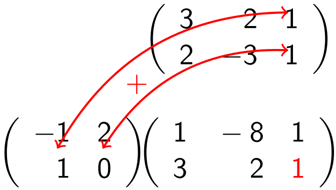
$$\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -8 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

$$1 \times 2 - 0 \times 3 = 2$$

Visual example

If $A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$, then

$C = A \times B = \begin{pmatrix} 1 & -8 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ which can be found as :



$$1 \times 1 + 0 \times 1 = 1$$

Let's have :

$$C = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 5 & 8 \end{pmatrix}, D = \begin{pmatrix} 7 & 6 \\ 10 & 9 \end{pmatrix}$$

We cannot calculate $C \times D$ but

$$D \times C = \begin{pmatrix} 7 \times 2 + 6 \times 1 & 7 \times 3 + 6 \times 5 & 7 \times 4 + 6 \times 8 \\ 10 \times 2 + 9 \times 1 & 10 \times 3 + 9 \times 5 & 10 \times 4 + 9 \times 8 \end{pmatrix}$$

and

$$D \times C = \begin{pmatrix} 20 & 51 & 76 \\ 29 & 75 & 112 \end{pmatrix}$$

Let's have :

$$E = \begin{pmatrix} -7 & 3 \\ 2 & 8 \\ 5 & 1 \end{pmatrix}, F = \begin{pmatrix} -8 & -4 \\ -6 & 5 \end{pmatrix}$$

and then :

$$E \times F = \begin{pmatrix} 38 & 43 \\ -64 & 32 \\ -46 & -15 \end{pmatrix}$$

Your turn to calculate a product !

Let's have :

$$P = \begin{pmatrix} -10 & 1 \\ -9 & -4 \end{pmatrix}, Q = \begin{pmatrix} 0 & 9 & 8 \\ -3 & -5 & -4 \end{pmatrix}$$

and then :

$$P \times Q = \begin{pmatrix} & & \end{pmatrix}$$

Your turn to calculate a product !

Let's have :

$$P = \begin{pmatrix} -10 & 1 \\ -9 & -4 \end{pmatrix}, Q = \begin{pmatrix} 0 & 9 & 8 \\ -3 & -5 & -4 \end{pmatrix}$$

and then :

$$P \times Q = \begin{pmatrix} -3 & & \\ & & \end{pmatrix}$$

Your turn to calculate a product !

Let's have :

$$P = \begin{pmatrix} -10 & 1 \\ -9 & -4 \end{pmatrix}, Q = \begin{pmatrix} 0 & 9 & 8 \\ -3 & -5 & -4 \end{pmatrix}$$

and then :

$$P \times Q = \begin{pmatrix} -3 & -95 & \\ & & \end{pmatrix}$$

Your turn to calculate a product !

Let's have :

$$P = \begin{pmatrix} -10 & 1 \\ -9 & -4 \end{pmatrix}, Q = \begin{pmatrix} 0 & 9 & 8 \\ -3 & -5 & -4 \end{pmatrix}$$

and then :

$$P \times Q = \begin{pmatrix} -3 & -95 & -84 \\ & & \end{pmatrix}$$

Your turn to calculate a product !

Let's have :

$$P = \begin{pmatrix} -10 & 1 \\ -9 & -4 \end{pmatrix}, Q = \begin{pmatrix} 0 & 9 & 8 \\ -3 & -5 & -4 \end{pmatrix}$$

and then :

$$P \times Q = \begin{pmatrix} -3 & -95 & -84 \\ -2 & & \end{pmatrix}$$

Your turn to calculate a product !

Let's have :

$$P = \begin{pmatrix} -10 & 1 \\ -9 & -4 \end{pmatrix}, Q = \begin{pmatrix} 0 & 9 & 8 \\ -3 & -5 & -4 \end{pmatrix}$$

and then :

$$P \times Q = \begin{pmatrix} -3 & -95 & -84 \\ -2 & -61 & \end{pmatrix}$$

Your turn to calculate a product !

Let's have :

$$P = \begin{pmatrix} -10 & 1 \\ -9 & -4 \end{pmatrix}, Q = \begin{pmatrix} 0 & 9 & 8 \\ -3 & -5 & -4 \end{pmatrix}$$

and then :

$$P \times Q = \begin{pmatrix} -3 & -95 & -84 \\ -2 & -61 & -56 \end{pmatrix}$$

A nerd store

Suppose a game store sells four types of product : Video Games (for a price of 50$ each), Comics (for $ each), Magic cards (for 4$ per pack) and candy bars (a bit of chocolate cost 1$).

product	price	
Video games	50$	per game
Comics	12$	per book
Magic cards	4$	per pack
Candy bars	1$	per bar

A nerd store

And let's suppose they are open five days per week : Monday to Friday. Here is the typical sell for one week :

	Video Games	Comics	Magic Cards	Candy Bars
Monday	5	33	55	201
Tuesday	4	42	20	192
Wednesday	8	55	25	212
Thursday	2	18	22	181
Friday	6	45	75	221

sales

$$
\begin{aligned}
\text{sales Monday} &= 5 \times 50\$ + 33 \times 12\$ + 55 \times 4\$ + 201 \times 1\$ \\
&= 1067\$ \\
\text{sales Tuesday} &= 4 \times 50\$ + 42 \times 12\$ + 20 \times 4\$ + 192 \times 1\$ \\
&= 976\$ \\
\text{sales Wednesday} &= 8 \times 50\$ + 55 \times 12\$ + 25 \times 4\$ + 212 \times 1\$ \\
&= 1372\$ \\
\text{sales Thursday} &= 2 \times 50\$ + 18 \times 12\$ + 22 \times 4\$ + 181 \times 1\$ \\
&= 585\$ \\
\text{sales Friday} &= 6 \times 50\$ + 45 \times 12\$ + 75 \times 4\$ + 221 \times 1\$ \\
&= 1361\$
\end{aligned}
$$

Do you recognize the pattern ?

sales

$$\begin{aligned}
\text{sales Monday} &= 5 \times 50\$ + 33 \times 12\$ + 55 \times 4\$ + 201 \times 1\$ \\
&= 1067\$ \\
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&= 585\$ \\
\text{sales Friday} &= 6 \times 50\$ + 45 \times 12\$ + 75 \times 4\$ + 221 \times 1\$ \\
&= 1361\$
\end{aligned}$$

Do you recognize the pattern ?

These formulas look like the formula for multiplying matrices !

So, if the "quantity" matrix is Q, and the prices are in the matrix P :

$$Q = \begin{pmatrix} 5 & 33 & 55 & 201 \\ 4 & 42 & 20 & 192 \\ 8 & 55 & 25 & 212 \\ 2 & 18 & 22 & 181 \\ 6 & 45 & 75 & 221 \end{pmatrix}, P = \begin{pmatrix} 50 \\ 12 \\ 4 \\ 1 \end{pmatrix}$$

Then the sales S are simply $s = Q \times p$.

$$S = Q \times p = \begin{pmatrix} 5 \times 50 + 33 \times 12 + 55 \times 4 + 201 \times 1 & = 106 \\ 4 \times 50 + 42 \times 12 + 20 \times 4 + 192 \times 1 & = 976 \\ 8 \times 50 + 55 \times 12 + 25 \times 4 + 212 \times 1 & = 1372 \\ 2 \times 50 + 18 \times 12 + 22 \times 4 + 181 \times 1 & = 585 \\ 6 \times 50 + 45 \times 12 + 75 \times 4 + 221 \times 1 & = 1361 \end{pmatrix}$$

The produce of matrices Q and P gives the sales!

Matrix algebra

- Addition is just summing elements
- Multiplication : remember the formula
- 0 matrix is filled with zeros
- 1 matrix has ones on its diagonal

Tomorrow

Why matrices are so powerful :

- We can solve equations, like with the sales !

For that we will see the

- determinant and minors
- the inverse of a matrix

Determinant

What is 1

In algebra, 1 is the number so

$$a \times 1 = 1 \times a = a$$

is there an equivalent of 1 for the matrices ?

What is 1

So we want a matrix I with, for any matrix A :

$$A \times I = I \times A = A$$

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and $I = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, then :

$$A \times I = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix} = A$$

What is 1

It means :

$$\begin{cases} ae + bg & = & a \\ af + bh & = & b \\ ce + dg & = & c \\ cf + dh & = & d \end{cases}$$

any idea of what should be the coefficients e, f, g and h ?

What is 1

The only general solution is a matrix with

- 1 on the diagonal
- 0 everywhere else

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ or, } I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix}.$$

I is called the identity matrix.